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COMBINING METHODS FOR SUBSURFACE PREDICTION

PETTER ABRAHAMSEN
Norwegian Computing Center, PO Box 114 Blindern, NO-0314 Oslo, Norway

Abstract. The depth to subsurfaces in a multi-layer model is obtained by adding the thickness of layers. However, the choice of layering is not unique so there will often be alternative ways of obtaining the depth to a particular subsurface. Each layer thickness can be described by a stochastic model accounting for uncertainties in the thickness. Stochastic models for the depth to subsurfaces are obtained from these. Alternative layer models will give alternative stochastic models and thus alternative depth predictions for the same subsurface. Two approaches to resolve this ambiguity is proposed. The first uses an established method of unbiased linear combination of predictors. The second and new approach combines the alternative stochastic models into a single stochastic model giving a single predictor for subsurface depth. This predictor performs similarly to the approach combining several predictors while drastically reducing computational costs. The proposed method applies to layered geological structures using a combination of universal or Bayesian kriging and cokriging.

1 Introduction

Consider the problem of mapping the depth to subsurfaces separating geological layers within a petroleum reservoir. The top and base of the reservoir are often visible on seismic data so accurate depth maps are obtained from depth converted travel time maps. The internal layering will rarely exhibit reliable seismic signals, so the thickness trend of each layer is mapped using geological interpretation of bore-hole data. The total thickness of the internal reservoir layers will seldom add up to the thickness depicted from seismic data. This ambiguity must be resolved to provide consistent depth maps describing the reservoir layers.

Two approaches for resolving this ambiguity is discussed. The first approach is adapted from econometrics and forecasting (Bunn, 1989; Granger, 1989), and consists of predicting the depth to the subsurfaces by combining alternative depth predictions 'in an optimal manner'. This approach works, but it is computationally expensive. An alternative and new approach is therefore proposed. Instead of combining the predictors, different stochastic models are combined. The result is a single stochastic model with a single associated predictor. These approaches perform very similar but computer expenses are dramatically reduced.

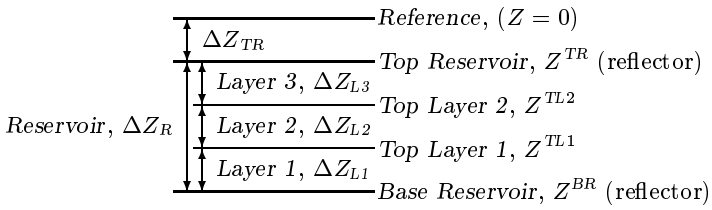

Figure 1. Schematic illustration of a reservoir formation. The double arrows indicate a stochastic model for the thickness of the corresponding layer, ΔZ_i.

2 Position of the problem

The stochastic model for the thickness of layer i may include a deterministic trend, $m_i(\mathbf{x})$, and a zero mean Gaussian random field, $\epsilon_i(\mathbf{x})$, for the residual error (Abrahamsen, 1993):

$$\Delta Z_i(\mathbf{x}) = m_i(\mathbf{x}) + \epsilon_i(\mathbf{x}); \qquad \mathbf{x} \in \mathbb{R}^2.$$

The stochastic model for the depth to subsurface l becomes $Z^l(\mathbf{x}) = \sum_{i=1}^{l} \Delta Z_i(\mathbf{x})$.

Figure 1 shows a schematic cross-section of a reservoir where subsurfaces *Top Reservoir* and *Base Reservoir* are assumed to be seismic reflectors. For a layer i bounded by two seismic reflectors, the trend is $m_i(\mathbf{x}) = v_i(\mathbf{x})\Delta t_i(\mathbf{x})$, where $v_i(\mathbf{x})$ is velocity and $\Delta t_i(\mathbf{x})$ is the seismic travel time. Models for the depth to *Top Reservoir* and *Base Reservoir* would be $Z^{TR}(\mathbf{x}) = \Delta Z_{TR}(\mathbf{x})$ and $Z^{BR}(\mathbf{x}) = \Delta Z_{TR}(\mathbf{x}) + \Delta Z_R(\mathbf{x})$ respectively (see Figure 1 for notation). Thickness trends, $m_i(\mathbf{x})$, for the internal reservoir layers are usually based on little data and many assumptions so the variance of the corresponding residual error could be large.

As an alternative method for obtaining the depth to *Base Reservoir* the thickness of all the internal layers could be added to *Top Reservoir*: $Z^{BR}(\mathbf{x}) = \Delta Z_{TR}(\mathbf{x}) + \Delta Z_{L3}(\mathbf{x}) + \Delta Z_{L2}(\mathbf{x}) + \Delta Z_{L1}(\mathbf{x})$. In practical applications the former model is preferred because seismic data are assumed more accurate than geological interpretation.

Lets look at a less obvious situation. The depth to *Top Layer 1* can be obtained by adding layer thicknesses to the depth of *Top Reservoir* or by subtracting layer thicknesses from the depth to *Base Reservoir*:

$$Z^{TL1}(\mathbf{x}) = \Delta Z_{TR}(\mathbf{x}) + \begin{cases} \Delta Z_{L2}(\mathbf{x}) + \Delta Z_{L3}(\mathbf{x}) & \text{add to } TR \\ \Delta Z_R(\mathbf{x}) - \Delta Z_{L1}(\mathbf{x}) & \text{subtract from } BR. \end{cases}$$

It is not obvious which alternative to choose. Since the seismic reflectors are assumed accurately determined, Figure 1 suggests that subtracting from *Base Reservoir* could be a better choice. Similar reasoning suggest that obtaining *Top Layer 2* by adding *Layer 3* to *Top Reservoir* is a good choice. However, these choices leaves a 'gap' between the two subsurfaces so the trend, $m_{L2}(\mathbf{x})$, for the thickness of *Layer 2* is never considered. Moreover, this choice has a serious implication on

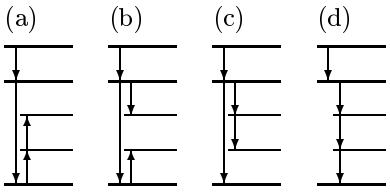

Figure 2. Four alternative methods for constructing the depth to the subsurfaces given in Figure 1.

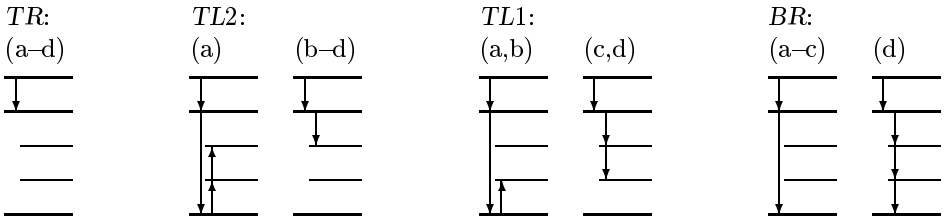

Figure 3. Alternative methods for constructing the depth to *Top Reservoir* (*TR*), *Top Layer 2* (*TL*2), *Top Layer 1* (*TL*1), and *Base Reservoir* (*BR*). Labels correspond to the graphs in Figure 2.

the uncertainty of the thickness of *Layer 2*: Assuming the residual errors, $\epsilon_i(\mathbf{x})$, to be independent implies that the variance of the thickness of *Layer 2* is

$$\mathrm{Var}\{Z^{TL2}(\mathbf{x}) - Z^{TL1}(\mathbf{x})\} = \mathrm{Var}\{\Delta Z_{L3}(\mathbf{x})\} + \mathrm{Var}\{\Delta Z_R(\mathbf{x})\} + \mathrm{Var}\{\Delta Z_{L1}(\mathbf{x})\}.$$

This variance is usually significantly larger than $\mathrm{Var}\{\Delta Z_{L2}(\mathbf{x})\}$ and the possible strong correlation between the depth to *Top Layer 1* and *Top Layer 2* is lost.

The discussion has motivated the need for an approach where several methods can be used simultaneously, so the unpleasant need for choosing one particular method becomes obsolete.

Figure 2 shows four graphs, each corresponding to a method for constructing *all* subsurfaces in Figure 1. The depth to a particular subsurface is found by following the arrows to the subsurface; an arrow pointing downwards means that the corresponding thickness is added whereas an arrow pointing upwards means that the corresponding thickness is subtracted. Although some graphs give the same result for a particular subsurface, the *dependencies* between the subsurfaces are different in all four graphs. This has an implication on the predictors for each subsurface (Abrahamsen, 1993). Thus, each graph in Figure 2 corresponds to a method for prediction of the set of subsurfaces so there are actually four different predictors for each of the four subsurfaces.

Figure 3 illustrates the methods for constructing subsurfaces slightly differently. Whereas, Figure 2 gives a method for *all* subsurfaces in each graph, Figure 3 shows the different methods for *each* subsurface. Each graph in Figure 3 are labelled using the labels in Figure 2. Note that although Figure 2 contains four graphs (or methods), there is only one or two possible ways of adding layers to obtain a particular subsurface.

3 Stochastic models for subsurfaces

A stochastic model with a linear trend for the thickness of layer i reads

$$\Delta Z_i(\mathbf{x}) = \underbrace{\mathbf{g}_i'(\mathbf{x})\,\boldsymbol{\beta}_i}_{m_i(\mathbf{x})} + \epsilon_i(\mathbf{x}); \qquad \mathbf{x} \in \mathbb{R}^2, \tag{1}$$

where $\mathbf{g}_i(\mathbf{x})$ is a vector of P_i known (deterministic) functions, $\boldsymbol{\beta}_i$ is a vector of P_i unknown parameters, and the residual error, $\epsilon_i(\mathbf{x})$, is a zero mean Gaussian random field specified by the standard error, $\sigma_i(\mathbf{x})$, and the correlation function $\rho_i(\mathbf{x}, \mathbf{y})$. The correlation function and standard error for the residual error can occasionally be estimated from bore-hole data. (1) is a multiple linear regression model with a correlated error term.

A typical model for the thickness of a layer i is $\Delta Z_i(\mathbf{x}) = \beta_{i1} + h(\mathbf{x})\,\beta_{i2} + \epsilon_i(\mathbf{x})$, where $h(\mathbf{x})$ is a trend supplied by geologists, so that $\mathbf{g}_i'(\mathbf{x}) = [1, h(\mathbf{x})]$. A typical travel time based model for the thickness of a layer i is $\Delta Z_i(\mathbf{x}) = [\beta_{i1} + \beta_{i2}\bar{t}_i(\mathbf{x})\,]\,\Delta t_i(\mathbf{x}) + \epsilon_i(\mathbf{x})$, where $\bar{t}_i(\mathbf{x})$ is the seismic travel time to the midpoint of layer i and $\Delta t_i(\mathbf{x})$ is the seismic travel time in layer i. So $\mathbf{g}_i'(\mathbf{x}) = \big[\Delta t_i(\mathbf{x}),\, \bar{t}_i(\mathbf{x})\Delta t_i(\mathbf{x})\big]$. A positive value for β_{i2} gives the widely encountered velocity increase at larger depthes due to compaction (Faust, 1951; Acheson, 1963). A similar velocity model was used by Hwang and McCorkindale (1994) for predicting the velocity field and by Xu, Tran, Srivastava and Journel (1992) for predicting depth. The residual error must account for the uncertainty in travel time (Walden and White, 1984; White, 1984) and the uncertainty in the interval velocity field (Al-Chalabi, 1974, 1979; Abrahamsen, 1993).

Consider now a multi-layer model including L layers and subsurfaces. The depth to the lth subsurface is

$$Z^l(\mathbf{x}) = \sum_{i=1}^{l} \Delta Z_i(\mathbf{x}) = \mathbf{f}^{l\,\prime}(\mathbf{x})\,\boldsymbol{\beta} + \mathcal{E}^l(\mathbf{x}),$$

where $\mathbf{f}^{l\,\prime}(\mathbf{x}) = \big[\mathbf{g}_1'(\mathbf{x}) \;\cdots\; \mathbf{g}_l'(\mathbf{x}) \;\; \mathbf{0}' \;\cdots\; \mathbf{0}'\big]$ and $\boldsymbol{\beta}' = \big[\boldsymbol{\beta}_1' \;\cdots\; \boldsymbol{\beta}_L'\big]$. Here, $\mathbf{0}$ are zero vectors replacing $\mathbf{g}_i(\mathbf{x})$ for $i = l+1, \ldots, L$, and $\mathcal{E}^l(\mathbf{x}) = \sum_{i=1}^{l} \epsilon_i(\mathbf{x})$.

4 Choice of Predictor

The best linear unbiased predictor for a random field with an unknown linear trend is the universal kriging predictor. All subsurfaces in a multilayer model are statistically dependent (covariates) since they all depend on the thickness of at least one common layer. So the kriging predictor for any subsurface should be conditioned on available depth observations from all correlated subsurfaces using universal cokriging (Abrahamsen, 1993).

The kriging predictors depend on the geometry of the observations, the choice of linear trends for the layer thicknesses, and the statistical properties of the residuals. So alternative methods, such as those illustrated in Figure 2, give different predictions for the same set of observations.

Universal kriging was used by Hwang and McCorkindale (1994) and cokriging by Jeffery, Stewart and Alexander (1996) for predicting velocity fields for depth conversion. Xu et al. (1992) finds that universal kriging and collocated cokriging give similar results for depth conversion. Using universal kriging however, gives the possibility of using non-linear relationships between depth and travel time.

5 Approaches to resolving the ambiguities

5.1 COMBINING PREDICTORS

This approach is an adaption of a method used in time series analysis and forecasting and is reviewed by Bunn (1989) and Granger (1989). The idea is to make a linear combination of alternative predictors.

For a subsurface l in Figure 1 there are four possible predictors corresponding to the four different graphs or methods in Figure 2: $Z^{*l}_{(\mathrm{a})}(\mathbf{x})$, $Z^{*l}_{(\mathrm{b})}(\mathbf{x})$, $Z^{*l}_{(\mathrm{c})}(\mathbf{x})$, and $Z^{*l}_{(\mathrm{d})}(\mathbf{x})$. A linear combination of these is a possible combined predictor:

$$\begin{aligned} Z^{*l}(\mathbf{x}) &= w^{*l}_{(\mathrm{a})}(\mathbf{x}) Z^{*l}_{(\mathrm{a})}(\mathbf{x}) + w^{*l}_{(\mathrm{b})}(\mathbf{x}) Z^{*l}_{(\mathrm{b})}(\mathbf{x}) + w^{*l}_{(\mathrm{c})}(\mathbf{x}) Z^{*l}_{(\mathrm{c})}(\mathbf{x}) + w^{*l}_{(\mathrm{d})}(\mathbf{x}) Z^{*l}_{(\mathrm{d})}(\mathbf{x}) \\ &= \boldsymbol{w}^{*l\prime}(\mathbf{x})\, \mathbf{Z}^{*l}(\mathbf{x}). \end{aligned} \tag{2}$$

Assume that each predictor is unbiased and that the covariance matrix, $\mathcal{C}^{*l}_{ab}(\mathbf{x}) = \mathrm{Cov}\left\{ Z^{*l}_a(\mathbf{x}) - Z^l_a(\mathbf{x}), Z^{*l}_b(\mathbf{x}) - Z^l_b(\mathbf{x}) \right\}$, of the predictors is known. Then, an unbiased predictor with the minimum prediction variance is obtained using weights

$$\boldsymbol{w}^{*l}(\mathbf{x}) = \boldsymbol{\mathcal{C}}^{*l\,-1}(\mathbf{x})\, \mathbf{e} / \left(\mathbf{e}' \boldsymbol{\mathcal{C}}^{*l\,-1}(\mathbf{x})\, \mathbf{e} \right), \tag{3}$$

where $\mathbf{e}$ is a vector of unit entries. This result is analogous to the weights obtained in ordinary kriging.

To form $\boldsymbol{\mathcal{C}}^*(\mathbf{x})$ requires the kriging prediction variances and even the prediction covariances between all predictors at any location $\mathbf{x}$. Thus, the drawback of this method is the necessity to evaluate several predictors, prediction variances, and prediction covariances for every subsurface.

5.2 COMBINING STOCHASTIC MODELS

This new approach propose that the alternative stochastic models for the depth to a particular subsurface should be combined according to the magnitude of the residual error for each model. A linear combination of the alternative stochastic models is considered.

There are two different methods and stochastic models for the depth to *Top Layer 1* in Figure 1 according to Figure 3. A linear combination reads

$$Z^{TL1}(\mathbf{x}) = w^{TL1}_{(\mathrm{a,b})}(\mathbf{x})\, Z^{TL1}_{(\mathrm{a,b})}(\mathbf{x}) + w^{TL1}_{(\mathrm{c,d})}(\mathbf{x})\, Z^{TL1}_{(\mathrm{c,d})}(\mathbf{x}). \tag{4}$$

The weights $w^{TL1}_{(\mathrm{a,b})}(\mathbf{x})$ and $w^{TL1}_{(\mathrm{c,d})}(\mathbf{x})$ are chosen to minimise the residual error variance of $Z^{TL1}(\mathbf{x})$ subject to the condition that the weights add to one:

$$\mathbf{w}^l(\mathbf{x}) = \mathbf{C}^{l\,-1}(\mathbf{x})\, \mathbf{e} / \left(\mathbf{e}' \mathbf{C}^{l\,-1}(\mathbf{x})\, \mathbf{e} \right), \tag{5}$$

where the elements of the covariance matrix, $C^l_{ab}(\mathbf{x}) = \mathrm{Cov}\{Z^l_a(\mathbf{x}), Z^l_b(\mathbf{x})\}$, are calculated using

$$\mathrm{Cov}\{Z^l_a(\mathbf{x}), Z^m_b(\mathbf{y})\} = \mathrm{Cov}\{\mathcal{E}^l_a(\mathbf{x}), \mathcal{E}^m_b(\mathbf{y})\} = \sum_{i \in \substack{\text{common} \\ \text{intervals}}} s^{ab}_i \,\mathrm{Cov}\{\epsilon_i(\mathbf{x}), \epsilon_i(\mathbf{y})\}, \quad (6)$$

where $s^{ab}_i = -1$ when interval i is added in one model and subtracted in the other. Otherwise, $s^{ab}_i = 1$. The combined residual error variance is $\mathrm{Var}\{Z^l(\mathbf{x})\} = [\mathbf{e}'\mathbf{C}^{l\,-1}(\mathbf{x})\,\mathbf{e}]^{-1}$ which is less than or equal to $\mathrm{Var}\{Z^l_a(\mathbf{x})\}$ for any method a.

Similar combinations must be constructed for all the subsurfaces. It is then straightforward — but requires some bookkeeping — to calculate covariances between depth observations from different subsurfaces. This leaves one stochastic model and a single associated predictor for the depth to any of the subsurfaces.

Combining predictors is based on the principle of minimising the prediction error. Combining stochastic models however, is a heuristic approach which must be justified by comparing the results to the results obtained when combining predictors. An example will illustrate that the two approaches give almost identical results. The advantage of the latter approach is speed because only one predictor for each subsurface is needed.

5.3 RELATED APPROACHES

(2) has the form of a multiple linear regression model for $Z^{*l}(\mathbf{x})$ with $Z^{*l}_a(\mathbf{x})$; a = (a), (b), (c), (d) as regressors and the weights as unknown parameters. A constant term accounting for possible bias can be added (Granger, 1989). This approach, called 'stacked regression' by Wolpert (1992) and Breiman (1992), either requires historical data or a large data set allowing cross validation. The review by Clemen (1989) compares different methods for choosing the weights in (2).

6 A Synthetic Example

6.1 STOCHASTIC MODELS

Consider the schematic cross section of a reservoir formation illustrated in Figure 1 and assume constant thickness for each reservoir zone:

$$\Delta Z_{Li}(x) = \beta_{Li} + \epsilon_{Li}(x); \qquad i = 1, 2, 3,$$

where $x \in \mathbb{R}$ since only a cross-section is considered. Moreover, $\Delta Z_{TR}(x)$ and $\Delta Z_R(x)$ are given as:

$$\Delta Z_{TR}(x) = \Big[\beta_{TR1} + \beta_{TR2}\left\{t_{TR}(x) - \mathrm{mean}\big(t_{TR}(x)\big)\right\}\Big] t_{TR}(x) + \epsilon_{TR}(x)$$

$$\Delta Z_R(x) = \left[\beta_{R1} + \beta_{R2}\frac{5-x}{10}\right] \Delta t_R(x) + \epsilon_R(x).$$

The expressions in the square brackets are the seismic velocities. A cross-section of the travel times is shown in Figure 4. A positive value for β_{TR2} gives the usual

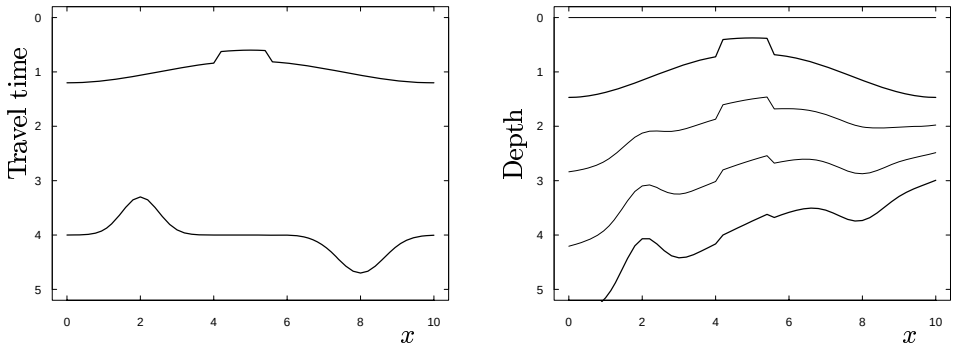

Figure 4. Travel times to *Top Reservoir* and *Base Reservoir* (left). Depth trends obtained when choosing all β's equal to one (right).

Table 1. Specified residual errors σ_a^l, calculated weights w_a^l, and combined residual errors σ^l. The methods correspond to labels in Figure 3. Note how the weights favour the assumed most accurate models.

Subsurface l	Method a	Res. error $\mathrm{Var}\{Z_a^l\}^{1/2}$	Weight w_a^l	Comb. res. error $\mathrm{Var}\{Z^l\}^{1/2}$
Top Reservoir:	(a–d)	0.1	1	0.1
Top Layer 2:	(a)	0.316	0.31	0.194
	(b–d)	0.224	0.69	
Top Layer 1:	(a,b)	0.245	0.62	0.202
	(c,d)	0.300	0.38	
Base Reservoir:	(a–c)	0.141	0.92	0.139
	(d)	0.361	0.08	

velocity increase with increasing depth causing the subsurfaces to be more curved than the travel times. A positive value for β_{R2} leads to a reduced interval velocity for higher x values causing *Base Reservoir* to tilt upwards towards the right. The standard errors of the residual errors for the layer thicknesses are chosen as $\sigma_{TR}(x) = \sigma_R(x) = 0.1$, $\sigma_{TL1}(x) = \sigma_{TL2}(x) = \sigma_{TL3}(x) = 0.2$, and they are assumed to be independent for simplicity in the example.

When combining models, the possible methods for constructing each of the subsurfaces are illustrated in Figure 3. The covariance matrix $\mathbf{C}^l$ has dimension one for *Top Reservoir* and dimension two for the three other subsurfaces. Note that $\mathbf{C}^l$ is independent of x because the standard errors of the residual errors are assumed constant. The resulting weights from (5) and residual errors of the combined models are given in Table 6.1.

Choosing all β-parameters equal to one and combining the trends according to (4) using the weights in Table 6.1, gives the depth trends illustrated in Figure 4. This set of trends are considered the 'truth' in the following.

6.2 SIMULATION EXPERIMENTS

Universal kriging predictors split into two parts; the estimated trend and the local fitting to observations (Cressie, 1993). The estimated trend depends heavily on the trend model while the local fitting is mainly dependent on the shape of the correlation function (variogram). The estimated trend will be used instead of the full kriging predictor because using trends will exaggerate potential problems and differences between approaches. Moreover, areas away from wells are the most difficult to predict accurately and therefore the areas of the greatest concern. The conclusions reached will carry over to the less sensitive kriging predictors in areas outside well control. It is reasonable to assume that results are valid in the vicinity of well observations. Three different approaches are tested:

1. Combining predictors (estimated trends) according to (2).
2. Estimating trends using stochastic models combined similar to (4).
3. Like 2. but using a Bayesian prior on the β's.

Ten sets of depth observations have been drawn from a multinormal distribution with the expectations given by the depth trends in Figure 4, and covariances obtained from the weights and (6). The location of these observations are obtained by dividing the x-axis into three segments and "drilling" one vertical well in each segment at a random location.

Trends have been estimated for each set of observations using the three approaches. The resulting ten sets of trends are seen in Figure 5.

The first approach combines four trend estimates (see Figure 2) using weights obtained from (3). Note that these weights depend on x.

The second approach, combining models, gives some trends that are far off the 'true trends' in Figure 4. This is caused by severe collinearity making it almost impossible to estimate some of the β-parameters.

The third approach, imposing a prior distribution on the β-parameters with expectations 0.5 and $\mathrm{Cov}\{\boldsymbol{\beta}\} = \mathrm{diag}(2)$, dramatically improves the estimates of the β-parameters. The corresponding ten trends in Figure 5 show a behaviour very similar to the one obtained by combining predictors. The prior distribution effectively restricts the parameter space so that extreme β estimates are prohibited. Choosing a prior with large standard error (200%) and an expectation far away from the true value (0.5 compared to 1) still gives good results. So the approach is appearently robust to poor and vague prior specifications.

6.2.1 *Bias and Errors*

To investigate bias and accuracy, one hundred sets of observations have been drawn using the procedure described above. Figure 6 (left) displays the average empirical bias (difference between 'true' and estimated trend) of the resulting hundred estimated trends for subsurface *Top Layer 2*. It is seen that all three approaches have little bias (<3%). This is expected since model assumptions for the estimators agree with the model that generated the data. However the average empirical trend error, $\mathrm{Var}\{\text{'true'} - \text{estimated trend}\}^{1/2}$, in Figure 6 (right) clearly show that the model combination approach have difficulties. The two other approaches produce acceptable empirical prediction errors.

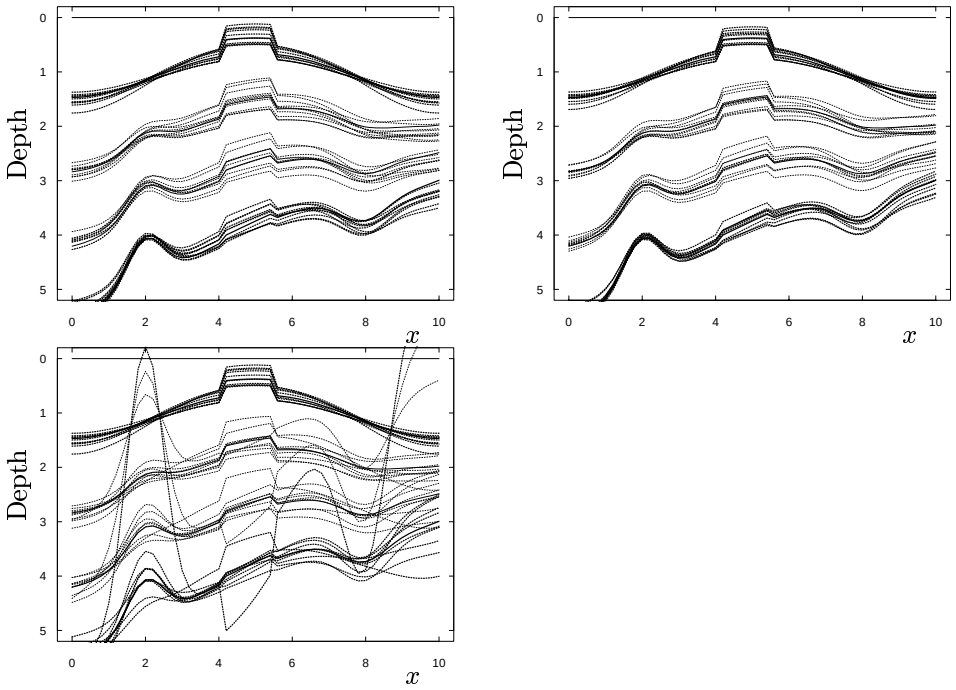

Figure 5. Ten sets of trends obtained by: 1. combining estimated trends (top left), 2. combining stochastic models (bottom left), and 3. combining stochastic models and using a Bayesian prior on the β's (top right).

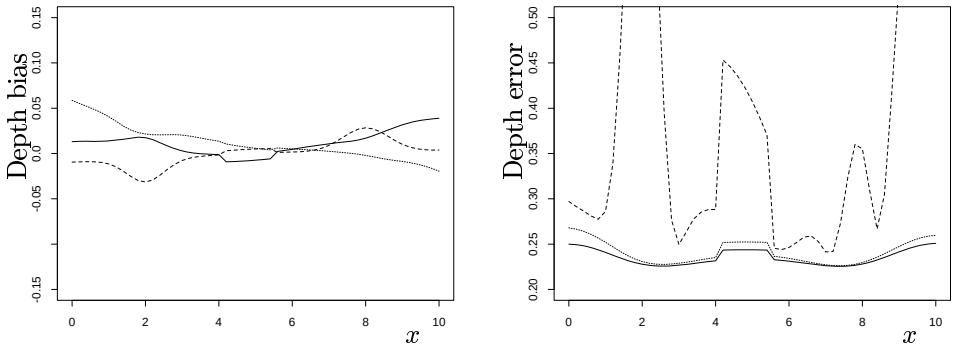

Figure 6. Average empirical bias (left) and error (right) from 100 simulations for *Top Layer 2*. ($\cdots$) combined predictors, (- - -) combined model, and (—) combined model with priors on β parameters.

7 Closing remarks

Two solutions to the problem of combining different methods for obtaining depthes to subsurfaces have been discussed. The first approach combines alternative pre-

dictors and the second approach merges alternative stochastic models. The latter approach is approximately 10 times faster but suffers from collinearities that are handled by imposing a prior distribution. The example showed that even a misspecified prior gave good results so the approach appear to be robust.

When combining predictors, a rigorous minimisation criteria for the prediction error is employed. The approach combining models however, uses a heuristic minimisation criteria for the residual variance. The usefulness of this approach is therefore justified by its performance. The two methods gave almost identical results for the synthetic example so in this situation it is possible to conclude that the model combination approach performs equally well.

The method has been implemented in commercial software and has been succesfully used in many field studies.

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