GEOSTATISTICS FOR SEISMIC DEPTH CONVERSION

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Abstract. Different kriging methods used for seismic depth conversion found in the literature is reviewed. It is concluded that universal kriging is superior to cokriging since it allows a better (non-linear) relationship between travel times and depth. An even better method is to use Bayesian cokriging for including correlations between different subsurfaces and interval velocity fields in a consistent way.

Contents

1. Introduction

Seismic depth conversion is the process of transforming interpreted seismic travel time maps to depth maps. Available data is:

- Depth observations in wells.
- Interpreted travel times.
- Interval velocities from check shots in wells.
- Interval velocities from stacking velocities.
- Interpreted interval velocities.
- General geophysical knowledge.

The objective is to integrate these data sources to obtain a description of the depth to subsurfaces with a measure of the uncertainty.

Kriging, which is a statistical method for spatial interpolation, provides different methods to accommodate this goal. This note reviews different kriging techniques and suggest the best approach.

2. Kriging

Kriging is a set of statistical techniques used for interpolation. They all seek to minimize the prediction error based on the assumption that the data are spatially correlated. The correlation is described by a *correlation function* (Abrahamsen 1994) or alternatively a *(semi)variogram*. The choice of correlation function determines the structure of the subsurface. Ideally the correlation function should be estimated from data but in most depth conversion applications the number of observations are to small (<50) so it has to be specified based on prior experience and sound judgment.

General references to kriging are Journel & Huijbregts (1978), Isaaks & Srivastava (1989), Cressie (1991), and Wackernagel (1995). For a historical view see Cressie (1990) who states that the French mathematician George Matheron (Matheron 1963) is the true father of kriging.

In this section the basic kriging techniques are outlined. The methods give a *predictor* for the depth, that is, a prediction of the most probable depth at any location (x, y), given observations. Also a *prediction error* which measures the uncertainty in the depth prediction is supplied.

2.1. Simple kriging. This is the basic method from which the other kriging approaches are extensions. The fundamental assumptions used in simple kriging is that the *expected value* or *trend* of the depth to a subsurface is completely known. The predicted depths will adjust to well observations, but far away from observations the prediction will coincidence with the specified trend. Therefore, this method is not recommended since the trend is never completely known.

2.2. Universal kriging. This approach relaxes the assumption of a completely known trend. Instead, a trend with unknown coefficients is used, e.g.

$$z(x,y) = \underbrace{a + b \cdot t(x,y)}_{\text{Trend}} + \epsilon(x,y)$$

where z is depth, t is travel time, and a and b are unknown coefficients to be estimated from well observations. The *residual error*, $\epsilon(x, y)$, is a spatially correlated random field with expectation zero. The residual error accounts for the random fluctuations of z around the trend. The trend can take any form but the unknown coefficients must enter linearly. Note that the trend model is a *linear multiple regression model.* The coefficients are determined by *generalized least squares* which is similar to a *least squares* approach where spatial correlations are considered. The universal kriging predictor is obtain by first estimating the trend, that is, the coefficients are estimated from data. Secondly, the estimated trend is considered the correct trend, and simple kriging is used for interpolating the well observations. The predictor will approach the estimated trend away from observations.

Universal kriging is unstable if the data carry little information on the value of the coefficients. This typically occurs if there are to many unknown coefficients in the trend compared to the number of data. This causes the prediction error to increase enormously away from the observations.

The correlation function for the residual error depends on the chosen trend. A complex trend including large scale structures should in general reduce the residual error and the correlation range. The estimated trend also depends on the correlation function. This circular dependency makes it difficult to estimate correlation functions (variograms) for universal kriging methods.

Universal kriging is a flexible method and is recommended despite the difficulties in determining the correlation function.

Universal kriging is often called *kriging with a trend* or *kriging with drift.*

2.3. Ordinary kriging. This method is universal kriging with a single unknown coefficient for the trend:

$$z(x, y) = a + \epsilon(x, y).$$

In most practical applications where ordinary kriging is used the data are abundant. This implies that only a subset of observations in a neighborhood is used instead of the complete dataset. This means that the initially constant trend is re-estimated at each location so a flexible and adaptive model arise.

For depth conversion applications ordinary kriging will seldom be appropriate since the number of data is to small to use a moving neighborhood of data.

2.4. Cokriging. Cokriging is used if there exist an additional source of data correlated to the primary variable of interest. E.g. travel time can be considered as correlated to depth. The basic assumption is that both variables have a known trend (usually constant) and the same correlation function. The correlation function can be different but there are severe restrictions on these choices so in practice the same correlation functions is used.

The two correlated variables are commonly called *covariates.* It is straight forward to extend cokriging to include several covariates.

It is possible to use trends with unknown coefficients, that is, to mix cokriging with universal kriging. Standard kriging packages seldom have this possibility.

A problem with cokriging when using a dense dataset such as travel times as the covariate is that it is impossible to use all the data. A remedy for this is to use only covariate data from a small neighborhood. The *collocated cokriging* approach (Xu, Tran, Srivastava & Journel 1992) takes this to the extreme by using only the single closest covariate data. data.

2.5. Bayesian kriging. Bayesian kriging is similar to universal kriging but there is an additional assumption that there exist *prior knowledge* on the value of the coefficients in the trend. This prior knowledge is specified as a prior multi Gaussian distribution for the parameters, so that expectation and variance must be specified. If the prior variance is zero, Bayesian kriging is identical to simple kriging. For large prior variances and/or many observations, Bayesian kriging approach universal kriging. Thus, simple and universal kriging can be considered as two extreme cases of Bayesian kriging.

Bayesian kriging is stable for any number of coefficients and data, including no well observations. This means that a flexible trend model can be used even though little well data exist.

3. Kriging in the geophysical literature

Although kriging is a flexible method for interpolating and extrapolating data in any dimension, the geophysical literature is sparse on kriging applications. Some exceptions are Olea (1974), Haas & Viallix (1976), Marechal (1984), Sprenke (1989), and Hansen (1993).

4. Seismic depth conversion

Some articles on depth conversion using kriging has been published. This is an attempt to comment on these approaches. Table 1 contains a list of methods and publications. I am not aware of any other published works using kriging for depth conversion.

Table 1. Published methods for seismic depth conversion using kriging techniques.

Author	Linear regression	Simple kriging	Ordinary kriging	Universal kriging	Cokriging	Bayesian kriging	Bayesian cokriging
Omre & Halvorsen (1989)		✓		✓		✓	
Omre, Halvorsen & Berteig (1989)		✓				✓	
Abrahamsen, Omre & Lia (1991)		✓		✓			✓
Abrahamsen (1993)		✓		✓			✓
Abrahamsen & Omre (1994)							✓
Xu et al. (1992)			✓	✓	✓ [a]		
Hwang & McCorkindale (1994)	✓		✓	✓			
Johnsen (1994)		✓			✓		

[a] Use collocated cokriging.

Some conclusions from these papers are:

- Kriging is superior to linear regression (Hwang & McCorkindale 1994).
- Universal kriging is superior to ordinary kriging (Xu et al. 1992, Hwang & McCorkindale 1994)

- Cokriging is superior to simple kriging (Johnsen 1994).
- Collocated cokriging and universal kriging performs equally good (Xu et al. 1992).

These conclusions can be summarized in the almost obvious statement:

> *Travel time data carry information and it should be included either as a trend or as a covariate using cokriging.*

4.1. Universal kriging or cokriging. The next question is whether cokriging or universal kriging should be used. Since both methods give similar results, the choice must be made on additional arguments. Universal kriging is far more flexible, and in my opinion, give better understanding of the relationship between depth and travel times. Below are some arguments supporting this view.

Computational speed: Universal kriging is in general faster since the number of observations used in the kriging algorithm only uses well observations. Speed however, is probably more dependent on efficient computer coding.

Variogram estimation: Using cokriging impose the same correlation function on both depth and travel times. This is hardly realistic. On the other hand, variograms in models with unknown trends are difficult to estimate. In many depth conversion applications however, the number of well data are to small for estimation of variograms and the user has to specify the using sound judgment anyway.

Missing travel time data: Universal kriging is not able to predict the depth at locations of missing travel time data whereas cokriging handles this properly. In my experience this has never been a problem in any case study, but this could be a serious disadvantage using universal kriging.

Physical relation between travel time and depth: Depth and travel time is ideally linked by the simple kinematic relation $z = v \cdot t$ where v is the interval velocity. The interval velocity is generally believed to increase with depth. A possible relation giving this behavior is $v = a + b \cdot t$ (Hwang & McCorkindale 1994). The corresponding depth model is $z = a \cdot t + b \cdot t^2$. This model is handled by universal kriging but not by cokriging. More general trends such as $v = a + b \cdot t + c \cdot x + d \cdot y$ has also proven useful on e.g. the Statfjord field. Cokriging can only model a linear relationship between z and t so velocity trends like the ones mentioned above can not be handled.

Stacking velocities: Universal kriging can also include interval velocities calculated from stacking velocities using Dix formula (Abrahamsen, Halvorsen & Omre 1989, Abrahamsen 1989). The bias in such velocities is adjusted for by adding an unknown constant which is estimated from well data. The uncertainty in the interval velocities can be accounted for using universal or Bayesian kriging

Conlusion:

> *Universal kriging is superior to cokriging for its flexibility in modeling the interval velocity.*

4.2. Bayesian kriging. A major problem with universal kriging is that the estimates of the coefficients are unstable when the number of well observations are few. Bayesian kriging (Kitanidis 1986, Omre 1987) solves this problem if there exist some prior knowledge on the coefficients in the model. For depth conversion this is truly the situation. Consider for instance the LINVEL model: $v = v_0 + k \cdot z$. It is experienced that $v_0 = 2000 \pm 500$m/s and that $k = 0.4 \pm 0.2s^{-1}$. Similar prior estimates on coefficients in interval velocity models are possible to specify. In Omre & Halvorsen (1989) it is demonstrated on Troll B-structure data how the prior guesses on the parameters are over ruled by the well data as more observations become available. Eventually, Bayesian kriging approach universal kriging as more data are introduced.

4.3. Bayesian cokriging. Abrahamsen et al. (1991) and Abrahamsen (1993) show how more than one subsurface in a layer-cake model can be predicted simultaneously using Bayesian cokriging. Using this method, well observations from subsurfaces above and below influence the depth prediction and the prediction error is reduced. In particular deviating wells are handled properly using this method. This approach has been tested on several North Sea fields including Gyda, Troll, Njord, Sleipner and Smørbukk South. Statoil has used this method extensively during the last years.

The Bayesian cokriging approach consider all subsurfaces and the interval velocity fields between them as covariates.

5. Velocity or depth prediction

Hwang & McCorkindale (1994) chose to predict interval velocities rather than depth directly. There are several reasons for *not* doing so:

- Depth prediction is the primary interest, interval velocities are only a link between the observed travel times and the depth.
- The uncertainty in the travel times are not possible to include when predicting interval velocities.
- It is impossible to translate depth observations to velocity observations at deviating wells correctly and automatically.

Within the Bayesian cokriging framework (Abrahamsen et al. 1991, Abrahamsen 1993) the interval velocity and the depth are considered as correlated random fields. This means that the kriging procedure can use interval velocity data (from check shots). The opposite is also possible: To predict interval velocity fields based on depth and interval velocity data.

6. Variogram estimation

The variogram or the spatial correlation function describes the shape or structure of the random component of the depth, that is, the difference between the true depth and the trend. The variogram therefore depends heavily on the trend.

The variogram is specified by the standard error[1], the correlation range, and finally some functional form such as 'spherical', 'exponential', or 'gaussian'. The user usually have to select a function and then try to match it to a plot of the empirical variogram

[1] The "sill' is often used for the variance which is the squared standard error.

by varying the correlation range and the standard error. This is difficult and totally unreliable for small datasets (<50). According to Webster & Oliver (1993), at least 100 data is needed.

Automatic procedures for variogram estimation exist but these need a lot of data to work properly. Therefore, be suspicious and use common sense.

For cokriging where the usual assumption is that there is a constant trend the correlation length will in general be very long (>10km). For a universal kriging model with a lot of structure modeled by the trend, the correlation length should be of the order 3km for depth conversion. A spherical correlation function proved the best for Base Cretaceous on Statfjord.

6.1. Anisotropy. Anisotropy means that the correlation length is different in two perpendicular directions. This means that two different correlation lengths must be estimated based on subsets of the original data. This exaggerates the problem of variogram estimation so anisotropic models are not recommended unless the number of observations is large.

Anisotropic variograms should never be used unless there are strong geological reasons for believing that such effects are important.

7. Conditional simulation

All kriging techniques have corresponding conditional simulation technique. Simulated realizations are mainly used for assessing volumetric uncertainties.

The Bayesian cokriging approach allows more data to be integrated so the overall variability of the simulated subsurfaces are reduced.

The average of a large set of simulations will approach the predicted depth obtained by kriging. Similarly the averaged squared deviations from the average will approach the squared prediction error. Hwang & McCorkindale (1994) use simulation to obtain a distribution for the depth at a location in the south-east region of the Troll field. A much simpler and faster approach is simply to calculate the prediction and prediction error at this point. The two approaches should give identical results.

8. Conclusion

Universal kriging is superior to cokriging for depth conversion since cokriging only allows a linear relationship between travel times and depth. See Section 4.1 for further arguments.

Bayesian kriging is superior to universal kriging when there are few well observations (<10). Bayesian kriging give similar results as universal kriging for many observations and can therefore always replace universal kriging.

Bayesian cokriging considers all intercorrelations between subsurfaces and interval velocity fields. I believe this is the best approach for depth prediction since all datatypes mentioned in the introduction is included and the differences in data quality is handled by this method.

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